

Remnants of light-cone propagation of correlations in dissipative systems

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We analyze the propagation of correlations after a sudden interaction change in a strongly interacting quantum system in contact with an environment. In particular, we consider an interaction quench in the Bose-Hubbard model, deep within the Mott-insulating phase, under the effect of dephasing. We observe that dissipation effectively speeds up the propagation of single-particle correlations while reducing their coherence. In contrast, for two-point density correlations, the initial ballistic propagation regime gives way to diffusion at intermediate times. Numerical simulations, based on a time-dependent matrix product state algorithm, are supplemented by a quantitatively accurate fermionic quasi-particle approach providing an intuitive description of the initial dynamics in terms of holon and doublon excitations.

In the last years, considerable experimental efforts have been devoted to dynamically generate complex states and monitor their evolution. Ultrafast optical pulses were used to photo-induce phase transitions in strongly interacting materials [1–4] while similar successes were reported for ultracold atoms using time-dependent electromagnetic fields [5, 6]. Despite these remarkable advances, the theoretical principles behind the non-equilibrium dynamics of strongly correlated matter are still far from being understood.

One of the salient questions relates to the propagation of correlations in complex systems. Providing an answer to this question would help understand how phase transitions can be dynamically induced. For example, to tune a system from a disordered to an ordered phase, such as into a superconducting or a Néel phase, one needs to establish how the correlations associated to a given order parameter build up. Understanding how correlations propagate following a perturbation would also provide major insights into the intricate mechanisms at play in complex systems.

Forays into the correlation dynamics of isolated systems have been made over the last decades. In a seminal work, Lieb and Robinson [7] demonstrated that for quantum spin systems described by local Hamiltonians the propagation of correlations is bound by an effective light-cone. This effect has for consequence that building up a given order parameter requires some time as the correlations need to set in. An elegant picture illustrating this effect was developed in [8] using conformal field theory and exactly solvable models. Within this picture, when a system is quenched to an Hamiltonian close to a quantum critical point, the propagation of correlations is carried by quasi-particle pairs generated during the quench. Experimental evidences for the light-cone effect was uncovered in cold atomic gases [9, 10], and in ionic systems characterized by long range interactions [11]. As first observed in [9], a bosonic gas was initially prepared deep within the Mott insulator phase by the application of an

optical lattice, then a sudden decrease of the intensity of the lattice potential brought the system slightly closer to the superfluid phase, and the subsequent evolution of the correlations was monitored and spatially resolved. Even for this non-integrable model, an approximate light-cone like propagation was found as shown in Fig. 1 (dashed line). Deep within the Mott insulator, the correlations were carried by the ballistic evolution of holons and doublons, quasi-particles corresponding to holes and excess particles within an atomic Mott insulator.

However, in most experimental setups, the system under study is coupled to an environment. Despite the ubiquitousness of environmental couplings, very few investigations have attempted to clarify their influence onto the dynamics of correlations. Works in this direction mainly focused on the behavior of correlations at long times showing, within the Markovian limit, that correlations decay on a length scale set by the Lieb-Robinson velocity and the system relaxation time [12, 13], and that an event horizon may emerge [14, 15]. Furthermore, in integrable models, the thermalization dynamics of correlations in the presence of white noise or starting from finite temperature initial states were explored in [16] and [17] respectively. While the environmentally-assisted diffusive emergence of long-lasting exotic pair correlations was investigated theoretically in [18]. Despite these recent findings, very little is known about the propagation of experimentally measurable correlations in dissipative strongly interacting systems.

To fill this gap, we focus here on the propagation of correlations in the one-dimensional Bose-Hubbard model following an abrupt interaction change while in contact with a memory-less environment which causes dephasing. We study this system both numerically, using a time-dependent matrix product state method (t-MPS) [19–21] for density operators [22, 23], and analytically describing the dynamics in terms of holon and doublon excitations, a description valid up to intermediate times for sufficiently large interaction strengths. We find that dissipation does

not affect all correlations equally. Single-particle correlations exhibit, up to intermediate times, a light-cone like propagation as in an isolated system. The presence of the dissipation effectively speeds up the propagation while reducing the coherence. In contrast, for the equal-time two-point density correlations the dissipative heating becomes quickly important and the initial ballistic propagation regime gives way to diffusion.

We study the dynamics of strongly interacting bosons in a one-dimensional lattice under the effect of local dephasing noise. The evolution of the density operator $\hat{\rho}$ is described by the master equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \mathcal{D}(\hat{\rho}). \quad (1)$$

The first term on the right-hand side describes the unitary evolution due to the Bose-Hubbard Hamiltonian

$$\hat{H} = -J \sum_j \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \text{H.c.} \right) + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1),$$

while the second term describes the dephasing noise in Lindblad form

$$\mathcal{D}(\hat{\rho}) = \gamma \sum_j \left(\hat{n}_j \hat{\rho} \hat{n}_j - \frac{1}{2} \{ \hat{n}_j^2, \hat{\rho} \} \right),$$

where \hat{a}_j^\dagger (\hat{a}_j) creates (annihilates) a boson at site j while $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$. Here J is the tunnelling constant, U the interaction strength, γ the dissipation strength, and $\{ \cdot, \cdot \}$ is the anti-commutator between two operators. The Bose-Hubbard Hamiltonian presents, at a critical ratio $(U/J)_c$ and for integer lattice fillings, a transition between a coherent superfluid phase, for $U/J < (U/J)_c$, and an incompressible Mott insulator phase for $U/J > (U/J)_c$ [24]. In one dimension and for constant filling $\bar{n} = \text{tr}(\hat{\rho} \hat{n}_j) = 1$, the transition occurs at a critical ratio $(U/J)_c \approx 3.4$ [25, 26].

The second term of Eq. (1), $\mathcal{D}(\hat{\rho})$, describes the effect of local dephasing, and, in the cold atom context, can be the dominant contribution arising from spontaneous emission from the optical lattice laser beams [27, 28]. This dissipator acts like a source of heat, generating excitations which increase local particle fluctuations and eventually would drive, within this model, the system towards the infinite temperature state. A stroboscopic application of the same quantum jump operator was shown to change the transport of particles from ballistic to diffusive as signalled by the evolution of local density [29].

We consider here a system at unit filling, $\bar{n} = 1$, initially prepared in the groundstate of the Bose-Hubbard model at $U/J \rightarrow \infty$, the atomic Mott insulator state. We investigate the propagation of correlations after an abrupt change of the interaction strength to a large but finite U/J value such that the corresponding groundstate would be Mott-insulating. As the interaction quench

takes place, the system is coupled to an environment and starts to undergo dephasing.

To gain a deeper analytical understanding of the initial dynamics, we reformulate the description of this system in terms of fermionic quasi-particles. For large final interaction strengths, this method was shown to provide accurate quantitative predictions and qualitative insights into the non-equilibrium dynamics of closed systems following a sudden quench [9, 30]. This approach is based on the realization that, for the isolated system and in the large interaction limit, the local occupation of a site ranges almost exclusively between $\bar{n} - 1$ and $\bar{n} + 1$. Hence, in one-dimension, making use of the Jordan-Wigner transformation, two types of auxiliary fermions can be introduced, $\hat{c}_{j,\pm}$, such that

$$\hat{a}_j^\dagger = \sqrt{\bar{n} + 1} Z_{j,+} \hat{c}_{j,+}^\dagger + \sqrt{\bar{n}} Z_{j,-} \hat{c}_{j,-}$$

where $Z_{j,+} = e^{i\pi \sum_{\sigma, l < j} \hat{n}_{l,\sigma}}$, $Z_{j,-} = Z_{j,+} e^{i\pi \hat{n}_{j,+}}$, $Z_{j,\pm}^\dagger = Z_{j,\pm}$ and the “+” and “-” fermions are doublons and holons respectively. The rewriting of excitations as fermions ensures that multiple doublons (or multiple holons) do not occupy the same site. The Bose-Hubbard Hamiltonian then reduces to

$$\begin{aligned} \hat{H}_f = \sum_j \mathcal{P} \Big[& -J(\bar{n} + 1) \hat{c}_{j,+}^\dagger \hat{c}_{j+1,+} - J\bar{n} \hat{c}_{j,-}^\dagger \hat{c}_{j+1,-} \\ & - J\sqrt{\bar{n}(\bar{n} + 1)} \left(\hat{c}_{j,+}^\dagger \hat{c}_{j+1,-}^\dagger - \hat{c}_{j,-} \hat{c}_{j+1,+} \right) \\ & + \frac{U}{4} (\hat{n}_{j,+} + \hat{n}_{j,-}) + \text{H.c.} \Big] \mathcal{P}, \end{aligned}$$

where the projector $\mathcal{P} = \prod_j (1 - \hat{n}_{j,+} \hat{n}_{j,-})$ ensures that a doublon and a holon do not occupy the same site. In the following, as was done in [9, 30], we set $\mathcal{P} = 1$ rendering the Hamiltonian \hat{H}_f quadratic and readily diagonalizable. However, the dissipator written in terms of fermionic quasi-particles, using $\hat{n}_j = \bar{n} + \hat{n}_{j,+} - \hat{n}_{j,-}$ and neglecting terms in $\hat{n}_{j,+} \hat{n}_{j,-}$ which are null in the presence of \mathcal{P} , is still not quadratic

$$\mathcal{D}_f(\hat{\rho}) = \gamma \sum_{j,\sigma=\pm} \left(\hat{n}_{j,\sigma} \hat{\rho} \hat{n}_{j,\sigma} - \frac{1}{2} \{ \hat{n}_{j,\sigma}, \hat{\rho} \} - \hat{n}_{j,\sigma} \hat{\rho} \hat{n}_{j,\bar{\sigma}} \right),$$

with $\bar{\sigma} = -\sigma$. Nevertheless, a closed set of differential equations can be obtained for the correlations $C_{+-}(d) = 1/L \sum_j \langle \hat{c}_{j+d,+} \hat{c}_{j,-} \rangle$ and $K(d) = 1/L \sum_j \langle \hat{c}_{j+d,+}^\dagger \hat{c}_{j,+} \rangle = 1/L \sum_j \langle \hat{c}_{j+d,-}^\dagger \hat{c}_{j,-} \rangle$. Due to the choice of the initial state, and as $K^*(d) = K(-d) = K(d)$ and $C_{+-}(d) =$

$-C_{+-}(-d)$, we get

$$\begin{aligned} \hbar \frac{d}{dt} C_{+-}(d) &= -\hbar\gamma C_{+-}(d) - iU C_{+-}(d) \\ &\quad - iJ\sqrt{\bar{n}(\bar{n}+1)}(\delta_{d,-1} - \delta_{d,1}) \\ &\quad - i2J\sqrt{\bar{n}(\bar{n}+1)}[K(d-1) - K(d+1)] \\ &\quad + iJ(1+2\bar{n})[C_{+-}(d-1) + C_{+-}(d+1)]; \\ \hbar \frac{d}{dt} K(d) &= -\hbar\gamma(1 - \delta_{d,0})K(d) \\ &\quad + iJ\sqrt{\bar{n}(\bar{n}+1)}[C_{+-}(d-1) - C_{+-}(d+1) \\ &\quad + C_{+-}^*(-d+1) - C_{+-}^*(-d-1)]. \end{aligned}$$

At $O(J/U)$, we obtain an explicit solution for $C_{+-}(d, t)$:

$$\begin{aligned} C_{+-}(d, t) &= \frac{J}{U} \left[\sqrt{\bar{n}(\bar{n}+1)} \frac{1 + i(\hbar\gamma/U)}{1 + (\hbar\gamma/U)^2} (\delta_{d,-1} - \delta_{d,1}) \right. \\ &\quad \left. + i(-1)^d \frac{\sqrt{\bar{n}(\bar{n}+1)}}{1 + (\hbar\gamma/U)^2} e^{-\gamma t} f(d, t) g(d, t) \right] \quad (2) \end{aligned}$$

where $f(d, t) = J_{d+1}(\tilde{J}t) + J_{d-1}(\tilde{J}t)$, with $\tilde{J} = 2(1 + 2\bar{n})J/\hbar$, J_n being the n^{th} order Bessel function of the first kind, and $g(d, t) = (\hbar\gamma/U - i) \sin(Ut/\hbar + \pi d/2) + (\hbar\gamma/U + i) \cos(Ut/\hbar + \pi d/2)$. In comparison, the leading term in the solution for $K(d, t)$ is of order $(J/U)^2$.

As shown in Fig. 1, for a dissipative coupling strength $\hbar\gamma = 0.4J$, the behavior of C_{+-} is non-trivially affected by the presence of an environment. The (blue) points are results from t-MPS simulations [31] while the (grey) full line, closely following these points, is the solution obtained within the quasi-particle picture. In the presence of dissipation, these correlations still propagate ballistically up to intermediate times while their coherence is reduced by $e^{-\gamma t}$ as illustrated by the envelope of the analytical solution, the (green) full line. However, as evidenced by the analytical expression, the effect of dissipation cannot be reduced to a simple exponential damping since the dissipation strength γ also enters through $g(d, t)$ as a prefactor to the oscillation term. In addition, the (red) solid line tracks the position of the first maximum of the envelope of C_{+-} for $\hbar\gamma = 0.4J$. For a given distance d , the first maximum appears at an earlier time compared to the $\gamma = 0$ case signalling an effective speed-up of the propagation of single-particle correlations. To support this statement, the time at which the first maximum reaches a given distance is reported for various dissipation strengths in the inset of Fig. 1. The observed increased velocity is a result of the interplay of the linear propagation of the envelope of C_{+-} and its exponential decay.

As discussed earlier, the dephasing does not affect all correlations equally. While, under both unitary and dissipative dynamics, the propagation of C_{+-} is approximately ballistic, the evolution of the two-point density

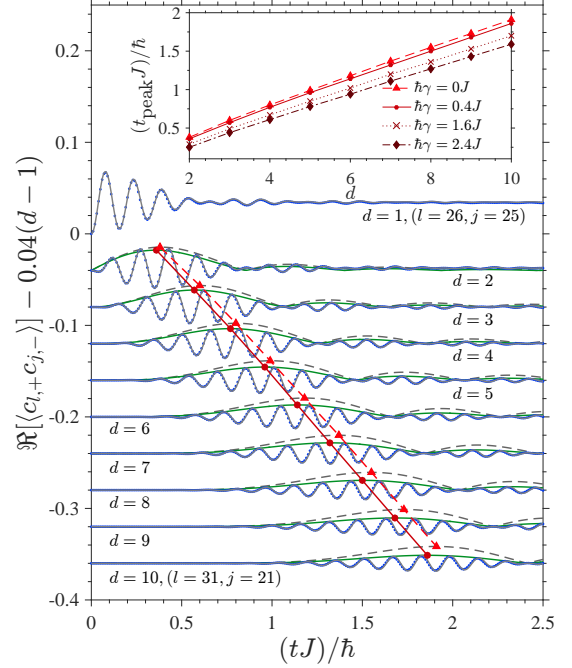


FIG. 1: (color online) Evolution of the real part of $C_{+-}(d, t)$ for various distances following a sudden interaction change from an atomic Mott insulator to $U = 40J$ in the presence of an environment with coupling $\hbar\gamma = 0.4J$ (the correlations are shifted by $-0.04(d-1)$ to improve readability). Full blue circles: t-MPS results; full grey line: analytical solution; full green line: envelope of the analytical solution; dashed grey line: envelope of the analytical solution for $\gamma = 0$; red circles and triangles: position of the first maximum of C_{+-} for $\hbar\gamma = 0.4J$ and 0 . Inset: time at which the first maximum of C_{+-} reaches a given distance for various dissipation strengths (obtained within the quasi-particle picture using Eq. (2)).

correlator is strongly altered by the coupling to an environment. Dissipation not only reduces coherence, but also heats up the system and increases number fluctuations leading to the proliferation of doublon-hole pairs and to the creation of even higher energy excitations corresponding to larger site occupancy. These effects are evidenced by the evolution of the density correlations plotted versus time for various dissipation strengths shown in Fig. 2. At short times, the propagation follows approximately the light-cone like regime, known from isolated systems, with an additional damping $e^{-2\gamma t}$. This is in agreement with results obtained from the quasi-particle picture. Within a mean-field decoupling of the density correlations, we obtain, to leading order in J/U , $\frac{1}{L} \sum_j \langle \hat{n}_{j+d} \hat{n}_j \rangle_{\text{MF}} \approx -2|C_{+-}(d)|^2$ for $d > 1$. However, for larger times, heating dominates the correlations and obliterates the light-cone structure. On Fig. 2 we indicate, using large dots, the times at which the first max-

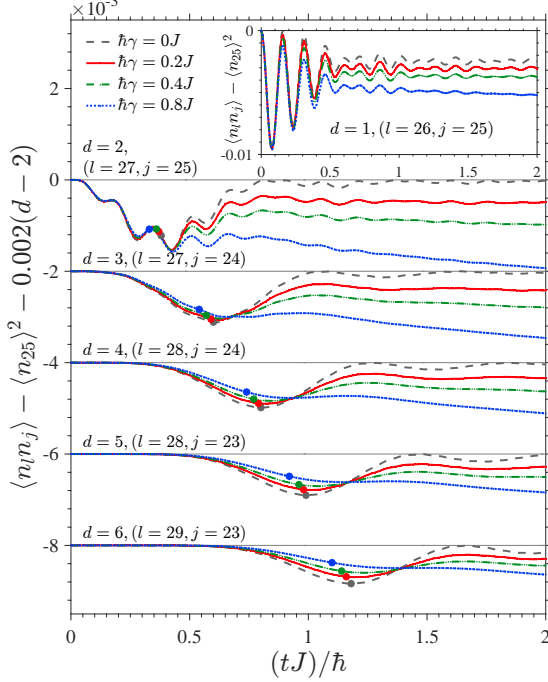


FIG. 2: (color online) Evolution of two-point density correlations following a sudden interaction change from an atomic Mott insulator to $U = 40J$ for various distances and dissipative strengths (the correlations are shifted by $-0.002(d-2)$). The large dots mark the times at which the first maximum of C_{+-} reaches a given distance.

imum of C_{+-} reaches a given distance. For $d > 2$, one sees that for $\gamma = 0$, the extrema of the single-particle and density correlations reach a given distance at the same time. For finite dissipation strengths the two extrema start to deviate as the maximum of the single-particle correlations effectively speeds up while the minimum of the density correlations shifts towards larger times. For sufficiently large γ or distances, this minimum is overwhelmed by the effect of heating rendering impossible its identification. Therefore, we find that at intermediate times the original ballistic propagation is replaced by a different propagation regime.

In order to understand this emerging regime, we plot in Fig. 3 these correlations as a function of distance for various times and two different dissipative strengths. For weak dissipation, $\hbar\gamma = 0.4J$, where the light-cone peak is still distinguishable, we find that after the passage of this maximum the correlations propagate, for $d > 2$, as $\exp(-\alpha(d-2)\sqrt{\hbar/(Jt)})$, where α is a free fitting parameter, hinting that the initial ballistic propagation has given way, at intermediate times, to diffusion. The emergence of this different propagation regime is also observed for larger γ as shown on the lower panel of Fig. 3. In this case, the light-cone peak is totally suppressed but one

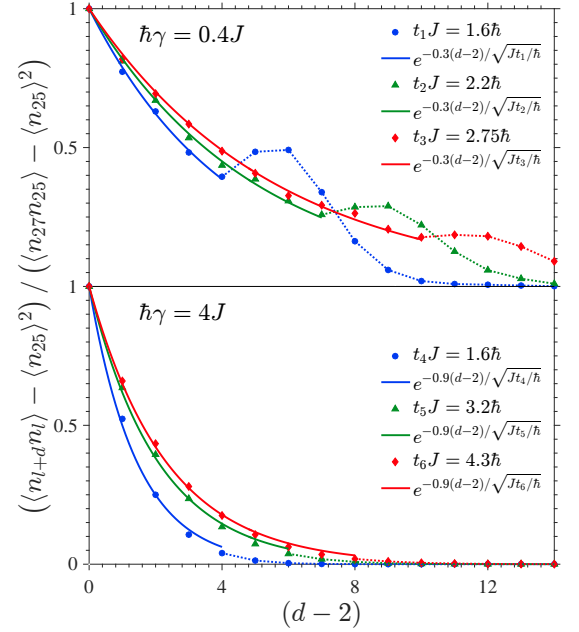


FIG. 3: (color online) Normalized density correlations as a function of distance for various times and two dissipative strengths. The full lines are fits to the function $\exp(-\alpha(d-2)\sqrt{\hbar/(Jt)})$, α is the only free parameter. The dashed lines are solely guides to the eye.

still sees that the size of the region over which correlations propagate diffusively increases as a function of time. The values of the fitting constant α agree for different times even though, at each time, it is extracted independently. Thus, α appears to be dependent mainly on the strength of the dissipative coupling.

To summarize, we analyzed the propagation of correlations in a strongly interacting Bose gas confined to an optical lattice after a sudden interaction change while the system is in contact with an environment. We found that the presence of dissipation does not affect the density and single-particle correlations in the same manner. Compared to the case of an isolated system, dissipation effectively speeds up the light-cone like propagation of single-particle correlations although their signal is suppressed over time. In contrast, for two-point density correlations, the initial ballistic propagation is rapidly overwhelmed by heating and enters a diffusive regime. The system studied here is experimentally realizable using the same state of the art experimental techniques developed in [9] to investigate an isolated quench, while only the dissipative coupling would need to be strengthened. This coupling could be enhanced by the application of an additional laser fluctuating both in space and time. We expect the effects uncovered in this study to be observ-

able over a wide regime of parameters and to arise in different systems with Markovian noise. Future studies might address how the propagation of correlations is affected by couplings to non-Markovian environments with a particular focus on solid state systems where, for example, phonons can constitute an important dissipative channel [4].

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 - [31] We simulate a one-dimensional lattice of 50 sites, with open boundary conditions. Locally we allow for an occupation of up to 4 particles and verify that the probability of having 4 particles on a given site is negligible. We simulate the system with a quantum number conserving code. We refer to the size of the auxiliary dimension of a MPS for each quantum number q as D_q . By considering $D_q \leq 80$ in each quantum number block, we keep a large bond dimension $D = \sum_q D_q$. Comparing the correlations obtained with $D_q = 80$ to $D_q = 40$ and $D_q = 60$, we find that these deviate by up to $5 \cdot 10^{-5}$ at long times. The evolution is implemented using a 4th order Suzuki-Trotter decomposition [32] with a time step $Jdt = 0.005\hbar$.
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